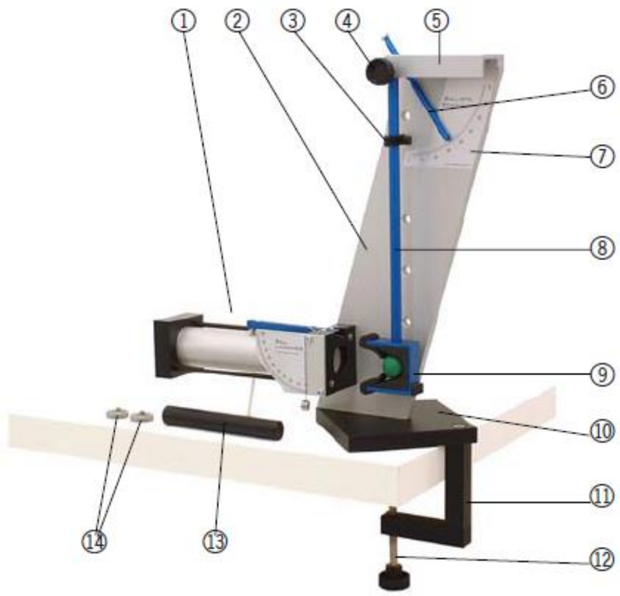
3B SCIENTIFIC® PHYSICS

1002656 Ballistic pendulum

Instruction sheet

12/15 MH



1 Projectile launcher 1002654
2 Back plate
3 Guide for swing pointer
4 Bearing screw
5 Counter bearing
6 Swing pointer
7 Angle scale
8 Pendulum
9 Projectile catcher
10 Base plate
11 Table clamp
12 Knurled screw
13 Ramrod (for 1002654)
14 Extra weights, 2 pcs.

Fig.1: Components

1. Safety instructions

- This instruction sheet is concerned mainly with the ballistic pendulum. You should also read the instructions for the projectile launcher 1002654.
- To check whether a projectile is located in the projectile launcher and the spring is cocked, only use the observation holes at the sides. Do not look into the barrel from the front. Risk of injury!
- Never aim at people!
- Protective goggles should be worn during the experiments.
- The projectile launcher should always be stored with the spring loose and with no projectile in the barrel.

2. Description

The ballistic pendulum is for experiment-based determination of the launch velocity of a projectile when it leaves the projectile launcher. It is also possible to determine trajectories when the projectile is launched horizontally or at an angle. Launch heights of 5, 10, 15, 20 or 30 cm can be selected easily with the aid of the drilled holes. Thanks to the extreme lightness of the pendulum, the experiment can be performed using comparatively safe plastic projectiles instead of steel balls. Experiments involving inelastic collisions (quantitatively) and elastic collisions (qualitatively) can be evaluated. The velocity of the projectiles determined from trajectory and pendulum experiments typically agree to within about 3%. Extra weights allow various pendulum travels to be investigated for constant speeds.

3. Operation and maintenance

- First the ballistic pendulum is screwed to a stable bench by means of its clamp. The projectile launcher is then screwed to the back plate (2) rom behind either in a horizontal position in front of the pendulum as in Fig. 1 or as shown in Fig 3.

  Tip: if the workbench is not stable enough, it may be that when the pendulum swings to its maximum extent and then swings back, it may jog the apparatus when striking the projectile launcher, causing the swing pointer to be shifted out of line. If this happens, the pendulum should rather be stopped by hand.
- Projectiles should always be loaded when the spring is not under tension by placing the sphere in loosely through the front of the plastic cylinder within the device. The sphere is then pushed down inside the barrel using the ramrod until the desired spring tension has been reached. The ramrod should not be removed too quickly, otherwise the suction its removal produces may pull the sphere out with it. The position of the sphere may only be checked using the observation holes. Never look into the barrel!
- Before launching, ensure that no one is in the way of the trajectory. To launch, the cord of the launching lever is briefly pulled perpendicularly to the lever.
- The pendulum (8) can be removed by undoing the bearing screw (4) and turned by 180° so that it is installed with the rear of the projectile catcher (9) pointing towards the launcher (experiments on elastic collision). The counter bearing (5) is designed so that the pendulum hangs at a slight angle if the bearing screw is only light tightened. This means that the projectile catcher is not precisely in front of the launch aperture of the launcher. For this reason, the bearing screw should be tightened until the catcher and the launch aperture are in line.
- After turning the pendulum round, or if necessary, the guide (3) for the swing pointer (6) should be adjusted so that the pointer just touches it when the pendulum is suspended at rest. The screw on the guide should only be finger-tightened to avoid the appearance of pressure on the pendulum rod.
- **Maintenance:** the ballistic pendulum principally requires no maintenance. If necessary some nonacidic grease (Vaseline) can be applied to the bearing screw (4) and the knurled screw (12). Other than in the vicinity of the scale, the apparatus may be cleaned using acetone, ethanol (white spirit) or petroleum ether as required. Avoid submerging the equipment in water.

4. Experiment procedure and evaluation

4.1 Ballistic pendulum

4.1.1. Experiment setup

- The experiment setup corresponds to Fig. 1 for experiments on inelastic collision. For experiments on elastic collisions, the pendulum should be turned round by 180° (cf. Section 3 “Operation”).

4.1.2. Experiment procedure

- It is practical for these experiments to enter the experiment number, the spring tension (1, 2 or 3), the type of collision (inelastic “i” or elastic “e”), the number of extra weights used and the measured angle φ. In order to obtain the most accurate experiment results, after one shot, a second should be performed with the swing pointer not having been reset to 0° in between. This minimizes the unavoidable frictional losses of the swing pointer. Example experiment sequence:

No	Spring tension	Type of collision	Extra weights	Angle φ
1	1	*p*	0	17.5
2	2	*p*	0	25.0
3	3	*p*	0	36.0
4	1	*p*	2	9.5
5	2	*p*	2	13.5
6	3	*p*	2	19.0
7	1	*e*	0	29.5
8	2	*e*	0	42.0
9	3	*e*	0	60.0

4.1.3. Experiment evaluation

4.1.3.1 Inelastic collision

The following equation is valid for the swinging pendulum due to conservation of energy

$$E_{pot} = E_{kin} \quad (1)$$

where the potential energy is

$$E_{pot} = m_{tot} \cdot g \cdot \Delta h \quad (2)$$

Here is m_{tot} the total mass of the pendulum including the projectile and any extra weights, g is the acceleration due to gravity and Δh is the difference in height of the center of gravity of the pendulum at rest and at the maximum extent of its swing.

From the measured angle φ and the measured length l_s, to the center of gravity according to Fig.2 the following is derived:

$$\Delta h = l_S \cdot (1 - \cos\varphi) \quad (3)$$

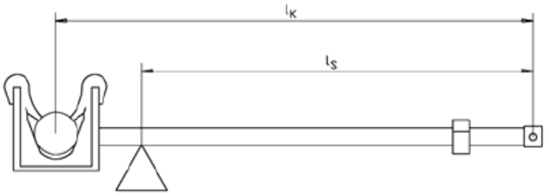


Fig. 2: Determining the required lengths. Distance between center of gravity and axis of rotation (l_s) should be measured including the projectile and any additional weights when the collision is inelastic. To perform the measurement, the pendulum may, for example, be balanced on a ruler mounted on its side. The distance between the center of the projectile and the axis of rotation is l_K = 280 mm.

The kinetic energy can be calculated from the moment of inertia I_{tot} relative to the axis of rotation and the maximum angular speed ω according to the equation:

$$E_{kin} = \frac{1}{2} \cdot I_{tot} \cdot \omega^2 \quad (4)$$

If Equations 2 and 4 are inserted into Equation 1 and Δh eliminated using Equation 3 then the equation can be rearranged to:

$$\omega = \sqrt{\frac{2 \cdot m_{tot} \cdot g \cdot l_s \cdot (1 - \cos\varphi)}{I_{tot}}} \quad (5)$$

However, we are not seeking ω, but the initial velocity of the projectile v_0. The relationship between the two values is given be the equation for the conservation of angular momentum directly before and after the collision:

$$L_K = L_{tot} \quad (6)$$

with the angular momentum of the projectile

$$L_K = m_K \cdot l_K \cdot v_0 \quad (7)$$

before the collision and the total angular momentum

$$L_{tot} = I_{tot} \cdot \omega \quad (8)$$

after the collision. Inserting Eqs. 7 and 8 into Eq. 6 gives

$$m_K \cdot l_K \cdot v_0 = I_{tot} \cdot \omega \quad (9)$$

Resolving this for ω and equating with Eq. 5 leads to the following relationship

$$v_0 = \frac{1}{m_K l_K} \cdot \sqrt{2 I_{tot} m_{tot} g l_s (1 - \cos\varphi)} \quad (10)$$

The moment of inertia is in principle determined from the integral

$$I_{tot} = \int_m l^2 dm \quad (11)$$

where l is the distance of each mass element from the axis of rotation. Since in this case it is not the moment of inertia that we seek to derive, I_{tot} can also be calculated from the period T of the pendulum (with projectile and any extra weights). For a physical pendulum the following is valid for small deflections:

$$I_{tot} = m_{tot} g l_s \left(\frac{T}{2\pi}\right)^2 \quad (12)$$

This means that all the variables are now known or calculable. For the above example, with m_K = 0.00695 kg the following table emerges:

No	**m_{tot} in kg**	**l_s in m**	**T in s**	**v_0 in m/s**
1	0.06295	0.218	1.01	3.39
2	0.06295	0.218	1.01	4.82
3	0.06295	0.218	1.01	6.88
4	0.09795	0.252	1.07	3.51
5	0.09795	0.252	1.07	4.98
6	0.09795	0.252	1.07	6.99

The numeric values should be determined separately for every pendulum, since material and manufacturing tolerances mean that values may differ from one to another.

4.1.3.2 Elastic collision

For a swinging pendulum Eq. 5 is still valid for the motion after a collision, the only difference being that the moment of inertia I_P is determined without the projectile but with any extra weights (pendulum mass m_P):

$$\omega = \sqrt{\frac{2 \cdot m_p \cdot g \cdot l_s \cdot (1-\cos\varphi)}{I_p}} \qquad (13)$$

To determine the relationship between ω and the initial velocity v_0 both the conservation of angular momentum and the conservation of energy before and after the collision must now be used. The additional equation is required since the system has an additional degree of freedom in the projectile velocity v_2 after the collision. As for Eq. 9, the following is true for the angular momentum:

$$m_K \cdot l_K \cdot v_0 = m_K \cdot l_K \cdot v_2 + I_p \cdot \omega$$
$$\Leftrightarrow \qquad (14)$$
$$v_2 = v_0 - \frac{I_p \cdot \omega}{m_K \cdot l_K}$$

If this velocity v_2 is inserted into the equation for the conservation of energy

$$\frac{1}{2} m_K \cdot v_0{}^2 = \frac{1}{2} m_K \cdot v_2{}^2 + \frac{1}{2} I_p \cdot \omega^2 \qquad (15)$$

by rearranging in various steps the following expression is obtained for v_0:

$$v_0 = \frac{1}{2}\omega l_K \left(1 + \frac{I_p}{m_K l_K{}^2}\right) \qquad (16)$$

If Eq. 13 is plugged in here and I_P determined as in Eq. 12, then v_0 can be calculated for an ideal inelastic collision. With m_K = 0.00695 kg:

N°	m_p in kg	l_s in m	T in s	v_0 in m/s
7	0.056	0.211	1.008	2.88
8	0.056	0.211	1.008	4.05
9	0.056	0.211	1.008	5.65

These values for v_0 are about 18% smaller than those obtained for inelastic collisions. This can be explained by the fact that the elastic collisions are not entirely ideal.

4.2 Determination of trajectories

4.2.1. Experiment setup

One possible experiment setup is shown schematically in Fig. 3 (not to scale). The drill holes in the back plate of the pendulum are placed so that when a projectile is fired to land directly on the workbench, the launch heights are 50, 100, 150, 200 and 300 mm.

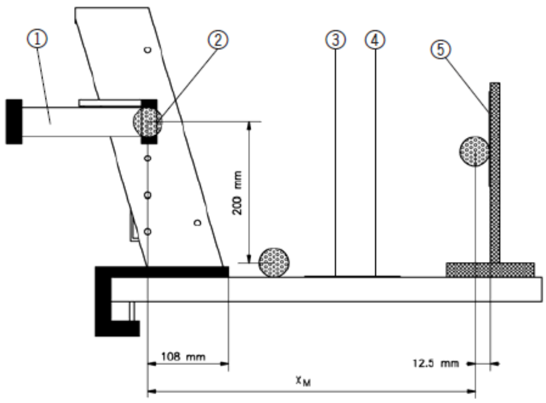


Fig. 3 Experiment setup, key: (1) Projectile launcher, (2) Launch position of projectile, (3) Paper, (4) Carbon paper, (5) Easel with whiteboard (for example)

When launching against a vertical wall the radius of the projectile (1.25 cm) should be subtracted from the distance between the point of launch and the wall to obtain the distance measurement x_M. The height measurement y_M relative to the launch height is given by the height of the impact on the wall minus 62.5 mm, 112.5 mm, 162.5 mm, 212.5 mm or 312.5 depending on the hole used.

4.2.2. Experiment procedure

It is practical for these experiments to note the experiment number, the spring tension (1, 2 or 3), the launch angle and the values x_M and y_M. Example with a launch angle $\varphi = 0°$:

No	Spring tension	Projectile distance x_M in cm	Target height y_M in cm
1	1	171.3	-30
2	2	125.4	-30
3	3	86.9	-30
4	1	62.3	-15
5	2	90.5	-15
6	3	120.7	-15

4.1.3. Experiment evaluation

It is practical to take as the origin of the coordinate system the mid-point of the projectile at the moment of launch. Then the following applies:

$$v_x = v_0 \cos\varphi \qquad (17)$$

$$v_y = v_0 \sin\varphi \qquad (18)$$

$$y = v_y t - \frac{1}{2} g t^2 \qquad (19)$$

$$x = v_x t \qquad (20)$$

From Eq. 20 $t = x / v_X$, whereby the time can be eliminated from Eq.19.

If v_X and v_Y are then eliminated from the resulting equation using Eqs. 17 and 18, the following is obtained

$$y = x \tan\varphi - x^2 \frac{g}{2 v_0^2 \cos^2\varphi} \qquad (21)$$

This is the equation for the trajectory.

In this equation only the launch velocity v_0 is unknown since the distances x and y where measured during the course of the experiments. If v_0 is calculated for the various experiments, the following results are obtained:

Spring tension	v_0 in m/s
1	3.53
2	5.10
3	6.85

The numbers are based on a total of 25 experiments, of which only 6 are explicitly listed in the above table. The trajectory can now be obtained from these using Eq. 21 and compared to the measured values. The result is shown in Fig. 4.

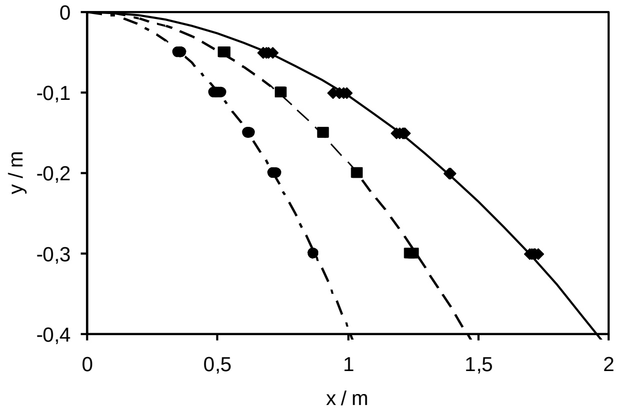


Fig. 4 Comparison of measurements and calculated curve, x = horizontal projectile distance, y = vertical height, symbols, measured values (circles = spring tension 1, squares = spring tension 2, rhombuses = spring tension 3), lines = calculated trajectories